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5 D Actions for 6 D Self-Dual Tensor Field Theory

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Abstract

We present two equivalent five-dimensional actions for six-dimensional $(N, 0)$ $N = 1, 2$ supersymmetric theories of self-dual tensor whose one spatial dimension is compactified on a circle. The Kaluza-Klein tower consists of a massless vector and infinite number of massive self-dual tensor multiplets living in five-dimensions. The self-duality follows from the equation of motion. Both actions are quadratic in field variables without any auxiliary field. When lifted back to six-dimensions, one of them gives a supersymmetric extension of the bosonic formulation for the chiral two-form tensor by Perry and Schwarz.

I. Introduction and Conclusion

One of the challenging problems in quantum field theories at present is to construct the action for chiral p-forms, i.e. anti-symmetric boson fields whose field strength is self-dual, especially with non-Abelian group structure implemented. Self-duality condition requires the spacetime to be Euclidean for odd p and Minkowskian for even p. In particular, $p = 2$ case has been of much attention related to the formulation of the world-volume action for M-theory five-brane [?, ?, ?, ?, ?, ?, ?].

Concerning Abelian theories of chiral p-forms there have been various types of proposals. Floreanini and Jackiw first proposed a non-manifestly covariant action for chiral scalars in two-dimensions by adopting somewhat unusual commutation relations among the field variables [?]. McClain, Wu and Yu proposed a formulation for chiral scalars by introducing an infinite number of auxiliary fields which do not carry any physical degree of freedom [?]. Each treatment was extended to higher order p-forms in Refs. [?] and [?, ?] respectively.

Two other formulations are also available. Pasti, Sorokin and Tonin introduced a Lorentz covariant formulation with only one auxiliary scalar field entering a chiral p-form action in a non-polynomial way [?]. Schwarz *et al.* studied a non-covariant formulation for self-dual two-form tensor in six-dimensions, where only five-dimensional Lorentz symmetry is manifest as one spacetime dimension is treated differently from the others [?, ?, ?]. However, it turned out that the PST formulation for $p = 2$ case contains local symmetries and a non-covariant gauge fixing of the local symmetries reduces to the non-manifestly Lorentz invariant formulation by Schwarz *et al.* [?, ?](see also [?, ?, ?]). Each formulation further developed to construct a kappa symmetric world-volume action for M-theory five-brane in an eleven-dimensional superspace background [?, ?].

The bosonic PST formulation was supersymmetrized in six-dimensions incorporating the self-dual tensor-multiplets. Dall'Agata *et al.* and Claus *et al.* presented the $(1, 0)$ and $(2, 0)$ supersymmetric extensions separately [?], [?]. On the other hand, the non-manifestly Lorentz invariant action by Schwarz *et al.* has not been supersymmetrized in the literature yet.

In the context of M-theory, five-dimensional maximally supersymmetric gauge theory at strong coupling limit is supposed to have description by a six-dimensional $(2, 0)$ fixed point, as the four-brane of type IIA theory is the M-theory five-brane wrapped around the eleventh direction and at strong coupling the eleventh dimension decompactifies developing an extra dimension [?, ?]. In fact, there is no interacting fixed point of the renormalization group in five-dimensions [?]. Nevertheless, direct field theoretic understanding of the relationship between the five and six dimensional theories for non-Abelian interactions is still lacking.

In this paper, we present two different but equivalent five-dimensional supersymmetric actions for the Kaluza-Klein modes of the six-dimensional $(N, 0)$, $N = 1, 2$ self-dual tensor-multiplets compactified on a circle. The Kaluza-Klein tower consists of a massless vector and infinite number of massive self-dual tensor multiplets living in five-dimensions. The self-duality follows from the equation of motion rather than a constraint imposed by hand. When lifted back to six-dimensions, one of our formulations gives a supersymmetric extension of the bosonic action for the chiral two-form tensor by Perry and Schwarz [?]. As there appears a five-dimensional vector multiplet after compactifying the 6 D tensor-multiplet on a circle, one may try to implement the non-Abelian group structure by taking the vector field as the usual Yang-Mills gauge field.[1] This would give a five-dimensional super Yang-Mills theory coupled with massive tensor-multiplets in an adjoint representation, realizing the M-theory picture on 5 D and 6 D theories. This scenario is the main motivation of the work in the present paper. The proposed formulations deal with Abelian case. Supersymmetry is provided, and non-Abelian generalization is to be done.

In the following section II., we first compactify the 6 D tensor-multiplets on a circle. The self-duality is expressed in terms of the Kaluza-Klein modes in five-dimensional language. The resulting Kaluza-Klein modes are massless vector and massive tensor multiplets which are identified by the analysis on five-dimensional supersymmetry algebra. In section III. and IV. we write our two proposed actions for the Kaluza-Klein modes of the $(1, 0)$ and $(2, 0)$ tensor-multiplets respectively. In section V. we lift the actions to six-dimensions and discuss the symmetries.

II. Tensor-multiplets Compactified on a Circle

Using the 4×4 gamma matrices, γ^μ, $\mu = 0, 1, \cdots, 4$, in five-dimensional Minkowskian spacetime with the metric, $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, \cdots, -1)$, the six-dimensional gamma matrices, $\Gamma^{\hat{\mu}}$, $\hat{\mu} = \mu, 5$, are taken here as

$$\Gamma^{\hat{\mu}} = \begin{pmatrix} 0 & \gamma^{\hat{\mu}} \\ \tilde{\gamma}^{\hat{\mu}} & 0 \end{pmatrix}, \qquad \gamma^\mu = \tilde{\gamma}^\mu, \quad \gamma^5 = -\tilde{\gamma}^5 = 1\,. \tag{II.1}$$

[1] In this approach, one needs to ensure the six-dimensional covariance of the non-Abelian gauge symmetry in the five-dimensional action.

This choice of gamma matrices gives a diagonalized Γ^7 matrix so that the non-vanishing components of the six-dimensional chiral spinors are upper four, ψ, only and the pseudo-Majorana *or* symplectic sp(N)-Majorana condition for 6 D $(N,0)$ chiral spinors is readily translated into the five-dimensional pseudo-Majorana condition [**?**]

$$\bar{\psi}_i = \psi^{i\dagger}\gamma^0 = \psi^{jt} C J_{ji} \,, \qquad\qquad J_{ij} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \,, \tag{II.2}$$

where $1 \leq i,j \leq 2N$ and C is the five-dimensional charge conjugate matrix satisfying $\gamma^{\mu t} = C\gamma^{\mu}C^{-1}\,, C^t = -C$.

6 D $(N,0)$, $N = 1,2$ tensor-multiplet consists of a two-form tensor, $B_{\hat{\mu}\hat{\nu}}$, pseudo-Majorana chiral spinors, ψ^i, and one for $N=1$/five for $N=2$ real scalar(s), ϕ [**?**].

Compactifying the fifth spatial dimension on a circle of radius R gives a Kaluza-Klein tower of the tensor-multiplets

$$\begin{aligned} B_{\hat{\mu}\hat{\nu}} &= \sum_{m\in\mathbb{Z}} B_{m\hat{\mu}\hat{\nu}}\, e^{i\frac{2\pi}{R}mx^5} \,, \qquad\qquad & \phi &= \sum_{m\in\mathbb{Z}} \phi_m\, e^{i\frac{2\pi}{R}mx^5} \,, \\ \psi^i &= \sum_{m\in\mathbb{Z}} \psi^i_m\, e^{i\frac{2\pi}{R}mx^5} \,, & \bar{\psi}_i &= \sum_{m\in\mathbb{Z}} \bar{\psi}_{mi}\, e^{i\frac{2\pi}{R}mx^5} \,. \end{aligned} \tag{II.3}$$

Reality and Pseudo-Majorana conditions imply

$$B^*_{m\hat{\mu}\hat{\nu}} = B_{-m\hat{\mu}\hat{\nu}} \,, \qquad \phi^*_m = \phi_{-m} \,, \qquad \bar{\psi}_{mi} = \psi^{i\dagger}_{-m}\gamma^0 = \psi^{j}_{m}{}^{t} C J_{ji} \,. \tag{II.4}$$

The self-duality of the 6 D two-form tensor, $H = *H$, is now expressed in terms of the five-dimensional Kaluza-Klein modes

$$F_{m\mu\nu} + i\tfrac{2\pi}{R} m B_{m\mu\nu} = \tfrac{1}{6}\epsilon_{\mu\nu}{}^{\lambda\rho\sigma} H_{m\lambda\rho\sigma} \,, \tag{II.5}$$

where $F_{m\mu\nu}$ is the field strength of $B_{m\mu 5} \equiv A_{m\mu}$.

Taking a curl of eq.(II.5) eliminates $A_{m\mu}$ leaving a second order differential equation that involves $B_{m\mu\nu}$ only

$$\partial_\lambda H^{\lambda\mu\nu}_m = i\tfrac{\pi}{3R} m\, \epsilon^{\mu\nu}{}_{\lambda\rho\sigma} H^{\lambda\rho\sigma}_m \,. \tag{II.6}$$

Reversely, taking off the curl, eq.(II.6) implies $\frac{1}{6}\epsilon_{\mu\nu}{}^{\lambda\rho\sigma}H_{m\lambda\rho\sigma} - i\frac{2\pi}{R}mB_{m\mu\nu} = F'_{m\mu\nu}$ for a certain $F'_{m\mu\nu} = \partial_\mu A'_{m\nu} - \partial_\nu A'_{m\mu}$. In $m \neq 0$ cases one can fix the gauge for the two-form tensor such that $F'_{m\mu\nu} = F_{m\mu\nu}$, while $m = 0$ case in eqs.(II.5,II.6) shows the hodge dual relation between the five-dimensional free Maxwell theory and a massless free two-form field theory. Thus, eq.(II.6) is equivalent to eq.(II.5) upto gauge transformations.

Six-dimensional $(N, 0)$ supersymmetry algebra naturally descends to five-dimensions

$$\{Q^i, \bar{Q}_j\} = \delta^i{}_j \gamma^{\hat{\mu}} P_{\hat{\mu}} = \delta^i{}_j (\gamma^\mu P_\mu + M)\,, \tag{II.7}$$

where the supercharges, Q^i, $1 \leq i \leq 2N$, satisfy the pseudo-Majorana condition (II.2) resulting in $8N$ real components, and $M = P_5$ is a real central charge. In particular, since the 6 D tensor-multiplet is massless, $p^{\hat{\mu}} p_{\hat{\mu}} = 0$, each Kaluza-Klein mode must satisfy

$$p^\mu p_\mu = (\tfrac{2\pi}{R} m)^2\,, \tag{II.8}$$

so that M acts as a "mass" operator on mth Kaluza-Klein mode with eigenvalue $\frac{2\pi}{R}m$. Massless modes, $m = 0$, and massive modes, $m \neq 0$, fit into the representations of the little groups, $\mathrm{SO}(3) \times \mathrm{Sp}(N)$ and $\mathrm{Spin}(4) \times \mathrm{Sp}(N) \sim \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{Sp}(N)$, separately. From [**?**] (see also [**?**]) the relevant representations of the massless and massive modes are for $N = 1$

$$\begin{aligned} (2,1) \times 2^2 &= (3,1) + (1,1) + (2,2) && : \text{massless tensor, Maxwell,} \\ (2,1,1) \times 2^2 &= (3,1,1) + (1,1,1) + (2,1,2) && : \text{massive tensor}\,, \end{aligned} \tag{II.9}$$

and for $N = 2$

$$\begin{aligned} (1,1) \times 2^4 &= (3,1) + (1,5) + (2,4) && : \text{massless tensor, Maxwell,} \\ (1,1,1) \times 2^4 &= (3,1,1) + (1,1,5) + (2,1,4) && : \text{massive tensor}\,, \end{aligned} \tag{II.10}$$

where for the massless representations the tensor and Maxwell multiplets are hodge dual to each other.

III. $(1, 0)$ Theory

Our two proposed five-dimensional Lagrangians for the Kaluza-Klein tower of the 6 D $(1, 0)$ tensor-multiplet compactified on a circle are

$$\begin{aligned} \mathcal{L}_1 = \sum_{m \in \mathbb{Z}} \quad & -\tfrac{1}{4}(F_{m\mu\nu} + i\tfrac{2\pi}{R} m B_{m\mu\nu})(F^{\mu\nu}_{-m} - i\tfrac{2\pi}{R} m B^{\mu\nu}_{-m} - \tfrac{1}{6}\epsilon^{\mu\nu\lambda\rho\sigma} H_{-m\lambda\rho\sigma}) \\ & + \bar{\psi}_{-mi}(i\gamma^\mu \partial_\mu + \tfrac{2\pi}{R} m)\psi^i_m + \partial_\mu \phi_m \partial^\mu \phi_{-m} - (\tfrac{2\pi}{R} m)^2 \phi_m \phi_{-m}\,, \end{aligned} \tag{III.1}$$

and

$$\mathcal{L}_2 = \sum_{m\in\mathbb{Z}} \quad \tfrac{1}{12}H_{m\lambda\mu\nu}H^{\lambda\mu\nu}_{-m} - i\tfrac{\pi}{12R}m\epsilon^{\mu\nu\lambda\rho\sigma}B_{m\mu\nu}H_{-m\lambda\rho\sigma}$$
$$+\bar{\psi}_{-mi}(i\gamma^\mu\partial_\mu + \tfrac{2\pi}{R}m)\psi^i_m + \partial_\mu\phi_m\partial^\mu\phi_{-m} - (\tfrac{2\pi}{R}m)^2\phi_m\phi_{-m}\,, \tag{III.2}$$

of which the supersymmetry transformation rules are

$$\delta B_{m\mu\nu} = i\bar{\varepsilon}_i\gamma_{\mu\nu}\psi^i_m\,, \qquad \delta\phi_m = i\bar{\psi}_{mi}\varepsilon^i\,, \qquad \delta A_{m\mu} = i\bar{\varepsilon}_i\gamma_\mu\psi^i_m\,,$$
$$\delta\psi^i_m = \begin{cases} \Big((\gamma^\mu\partial_\mu + i\tfrac{2\pi}{R}m)\phi_m + \tfrac{1}{4}(F_{m\mu\nu} + i\tfrac{2\pi}{R}mB_{m\mu\nu})\gamma^{\mu\nu}\Big)\,\varepsilon^i & \text{for } \mathcal{L}_1 \\ \Big((\gamma^\mu\partial_\mu + i\tfrac{2\pi}{R}m)\phi_m + \tfrac{1}{12}H_{m\lambda\mu\nu}\gamma^{\lambda\mu\nu}\Big)\,\varepsilon^i & \text{for } \mathcal{L}_2 \end{cases}\,. \tag{III.3}$$

Note that these are compatible with the reality and pseudo-Majorana conditions (II.4) and the invariance of the action can be shown using $\gamma^{\lambda\mu\nu} = \frac{1}{2}\epsilon^{\lambda\mu\nu\rho\sigma}\gamma_{\rho\sigma}$ and

$$\bar{\varepsilon}_i\gamma_{\mu_1}\gamma_{\mu_2}\cdots\gamma_{\mu_n}\psi^i_m = -\bar{\psi}_{mi}\gamma_{\mu_n}\cdots\gamma_{\mu_2}\gamma_{\mu_1}\varepsilon^i = -(\bar{\varepsilon}_i\gamma_{\mu_1}\gamma_{\mu_2}\cdots\gamma_{\mu_n}\psi^i_{-m})^*\,. \tag{III.4}$$

The summation of the modes can be just over $|m|$ and $-|m|$ for any given $m \in \mathbb{Z}$, as this pair alone forms an irreducible representation of the supersymmetry transformations. Hence, we may mix $\mathcal{L}_1$ and $\mathcal{L}_2$. For example, we may replace the zero mode in $\mathcal{L}_2$ by the zero mode in $\mathcal{L}_1$ which will give the kinetic terms for both the vector and the tensor fields.

For $\mathcal{L}_1$, the equations of motion for $B_{m\mu\nu}$, $m \neq 0$ and $A_{0\mu}$ alone give the self-duality formula (II.5) for $m \neq 0$, $m = 0$ respectively. On the other hand, $B_{0\mu\nu}$ appears only as a total derivative in $\mathcal{L}_1$ not contributing the action, and the equation of motion for $A_{m\mu}$, $m \neq 0$ is nothing but the divergence of the self-duality formula, and hence not a new field equation. Note also that the zero modes correspond to a five-dimensional super Maxwell theory.

For $\mathcal{L}_2$, the equation of motion for $B_{m\mu\nu}$ is the curl of the self-duality (II.6) and the gauge freedom recovers the self-duality as we discussed in section II.

Our two Lagrangians, $\mathcal{L}_1$ and $\mathcal{L}_2$, are dual to each other through the following intermediate Lagrangian containing auxiliary two-form fields, $J_{m\mu\nu} = -J_{m\nu\mu}$,

$$\mathcal{L}_{1\leftrightarrow 2} = \sum_{m\in\mathbb{Z}} \tfrac{1}{4}J_{m\mu\nu}J^{\mu\nu}_{-m} + \tfrac{1}{2}(F_{m\mu\nu} + i\tfrac{2\pi}{R}mB_{m\mu\nu})(J^{\mu\nu}_{-m} + \tfrac{1}{12}\epsilon^{\mu\nu\lambda\rho\sigma}H_{-m\lambda\rho\sigma})\,. \tag{III.5}$$

The equations of motion for $J_{m\mu\nu}$, $B_{m\mu\nu}$ are $J_{m\mu\nu} = -(F_{m\mu\nu} + i\frac{2\pi}{R}mB_{m\mu\nu})$, $J_{m\mu\nu} = -\frac{1}{6}\epsilon_{\mu\nu\lambda\rho\sigma}H_m^{\lambda\rho\sigma}$ respectively. Depending on which expression of the auxiliary fields we choose to substitute, $\mathcal{L}_{1\leftrightarrow 2}$ gives either $\mathcal{L}_1$ or $\mathcal{L}_2$. This equivalence has an analogue in three-dimensions: the free theory of Maxwell and Chern-Simons terms is equivalent to the theory of Chern-Simons and Higgs terms.

IV. $(2,0)$ Theory

The $(2,0)$ tensor-multiplet contains five real scalars, ϕ^{ij}, $1 \le i,j \le 4$, satisfying

$$\phi^{ij} = -\phi^{ji}\,, \qquad\qquad \phi^{ij}J_{ij} = 0\,. \tag{IV.1}$$

With $\phi_{ij} = \phi^{kl}J_{ki}J_{lj}$ our proposed Lagrangians are

$$\begin{aligned}\mathcal{L}_1 = \sum_{m\in\mathbb{Z}} \quad & -\tfrac{1}{4}(F_{m\mu\nu} + i\tfrac{2\pi}{R}mB_{m\mu\nu})(F^{\mu\nu}_{-m} - i\tfrac{2\pi}{R}mB^{\mu\nu}_{-m} - \tfrac{1}{6}\epsilon^{\mu\nu\lambda\rho\sigma}H_{-m\lambda\rho\sigma}) \\ & +\bar{\psi}_{-mi}(i\gamma^\mu\partial_\mu + \tfrac{2\pi}{R}m)\psi^i_m + \partial_\mu\phi^{ij}_m\partial^\mu\phi_{-mij} - (\tfrac{2\pi}{R}m)^2\phi^{ij}_m\phi_{-mij}\,,\end{aligned} \tag{IV.2}$$

and

$$\begin{aligned}\mathcal{L}_2 = \sum_{m\in\mathbb{Z}} \quad & \tfrac{1}{12}H_{m\lambda\mu\nu}H^{\lambda\mu\nu}_{-m} - i\tfrac{\pi}{12R}m\epsilon^{\mu\nu\lambda\rho\sigma}B_{m\mu\nu}H_{-m\lambda\rho\sigma} \\ & +\bar{\psi}_{-mi}(i\gamma^\mu\partial_\mu + \tfrac{2\pi}{R}m)\psi^i_m + \partial_\mu\phi^{ij}_m\partial^\mu\phi_{-mij} - (\tfrac{2\pi}{R}m)^2\phi^{ij}_m\phi_{-mij}\,,\end{aligned} \tag{IV.3}$$

and the supersymmetry transformation rules are

$$\begin{aligned} &\delta B_{m\mu\nu} = i\bar{\varepsilon}_i\gamma_{\mu\nu}\psi^i_m\,, \qquad\qquad \delta A_{m\mu} = i\bar{\varepsilon}_i\gamma_\mu\psi^i_m\,, \\ &\delta\phi^{ij}_m = -i\tfrac{1}{2}(\bar{\psi}^i_m\varepsilon^j - \bar{\psi}^j_m\varepsilon^i + \tfrac{1}{2}J^{-1ij}\bar{\psi}_{mk}\varepsilon^k)\,, \\ &\delta\psi^i_m = \begin{cases} (\gamma^\mu\partial_\mu + i\frac{2\pi}{R}m)\phi^i_{mj}\varepsilon^j + \frac{1}{4}(F_{m\mu\nu} + i\frac{2\pi}{R}mB_{m\mu\nu})\gamma^{\mu\nu}\varepsilon^i & \text{for } \mathcal{L}_1 \\ (\gamma^\mu\partial_\mu + i\frac{2\pi}{R}m)\phi^i_{mj}\varepsilon^j + \frac{1}{12}H_{m\lambda\mu\nu}\gamma^{\lambda\mu\nu}\varepsilon^i & \text{for } \mathcal{L}_2 \end{cases}\,.\end{aligned} \tag{IV.4}$$

V. Lift to $6\,$D

It is straightforward to lift our proposed actions to six-dimensions. The scalar and spinor parts are the standard ones

$$\tfrac{1}{2\pi R}\int \mathrm{d}^6x\; i\bar{\psi}_i\tilde{\gamma}^{\hat{\mu}}\partial_{\hat{\mu}}\psi^i + \partial_{\hat{\mu}}\phi\partial^{\hat{\mu}}\phi\,. \tag{V.1}$$

The two-form tensor part leads for $\mathcal{L}_1$

$$\tfrac{1}{2\pi R}\int \mathrm{d}^6x\,\tfrac{1}{4}H_{5\mu\nu}H^{5\mu\nu}+\tfrac{1}{24}\epsilon^{\mu\nu\lambda\rho\sigma}\partial_5 B_{\mu\nu}H_{\lambda\rho\sigma}\,, \tag{V.2}$$

and for $\mathcal{L}_2$

$$\tfrac{1}{2\pi R}\int \mathrm{d}^6x\,\tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}-\tfrac{1}{24}\epsilon^{\mu\nu\lambda\rho\sigma}\partial_5 B_{\mu\nu}H_{\lambda\rho\sigma}\,. \tag{V.3}$$

The latter is identical to the action by Perry and Schwarz [?]. Hence, our work on $\mathcal{L}_2$ can be regarded as its $(N,0)$, $N=1,2$ supersymmetric extensions. The supersymmetry transformation rules can be easily read from eqs.(III.3,IV.4). Some analysis on the canonical dimensions of the fields show $R=g^2_{\mathrm{YM}}$, where g_{YM} is the five-dimensional coupling constant [?, **?**].

Both actions in eqs.(V.2,V.3) are manifestly invariant under the five-dimensional Lorentz transformations

$$\delta_{5\mathrm{D}}B_{\mu\nu}=\Lambda_\mu{}^\lambda B_{\lambda\nu}+\Lambda_\nu{}^\lambda B_{\mu\lambda}+\Lambda^{\lambda\rho}x_\lambda\partial_\rho B_{\mu\nu}\,. \tag{V.4}$$

On the other hand, it is not clear whether the actions are invariant under the rotations mixing the sixth direction and the other five, μ, directions. In [?] the authors found a transformation with five-dimensional vector parameters, Λ_μ,

$$\delta B_{\mu\nu}=\tfrac{1}{6}\Lambda{\cdot}x\epsilon_{\mu\nu\lambda\rho\sigma}H^{\lambda\rho\sigma}+x^5\Lambda{\cdot}\partial B_{\mu\nu}\,, \tag{V.5}$$

which leaves the action (V.3) invariant upto surface terms, and it was argued that this is the remaining Lorentz symmetry so that the action possesses the full six-dimensional Lorentz symmetry. However, in this case the transformation in eq.(V.5) lacks the usual distinction of “spin” and “orbital” parts of the Lorentz transformations as in eq.(V.4). Furthermore, the anticommutator of the transformations reads

$$[\delta_2,\delta_1]B_{\mu\nu}=(\Lambda_1{\cdot}x\Lambda_2^\lambda-\Lambda_2{\cdot}x\Lambda_1^\lambda)(H_{\lambda\mu\nu}+\partial_\lambda B_{\mu\nu})\,. \tag{V.6}$$

For the transformation in eq.(V.5) to be identified with the remaining Lorentz transformations, this must be interpreted as the five-dimensional Lorentz transformations (V.4) upto any possible gauge transformation. However, direct calculation shows that this is not the case, since with $\Lambda_{\mu\nu}=\Lambda_{1\mu}\Lambda_{2\nu}-(1\leftrightarrow 2)$

$$[\delta_2,\delta_1]H_{\lambda\mu\nu}=\delta_{5\mathrm{D}}H_{\lambda\mu\nu}+\Lambda^{\tau\kappa}x_\tau\partial_\kappa H_{\lambda\mu\nu}-3\Lambda_{\kappa[\lambda}\partial^\kappa B_{\mu\nu]}\,. \tag{V.7}$$

and the right hand side can not be rescaled into[2] $\delta_{5\mathrm{D}}H_{\lambda\mu\nu}$. Therefore, eq.(V.5) is a symmetry of the action which is not Lorentz symmetry even upto gauge transformations. Nevertheless, the formulation by Schwarz *et al.* is a certain non-covariant gauge fixing of the PST

[2]This is also impossible on shell contrary to the claim in Ref. [?].

formulation [?, ?, ?, ?], and the latter possesses the full 6 D Lorentz symmetry. Furthermore it was shown that the PST action for the two-form tensor supermultiplet enjoys the six-dimensional superconformal symmetry [?] as well as some nontrivial local symmetries [?, ?]. These results suggest that there is a hidden big symmetry in the action which combines those two symmetries and contains the transformation found by Perry and Schwarz (V.5). Note that Coleman-Mandula theorem on possible symmetries of field theories applies only for massive point-like particles [**?**] and 6 D two-form tensor theory is not the case, since it is massless conformal theory and the self-duality makes the distinction between the electric and the magnetic particles meaningless.

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